

# Optimisation Problems for Planning Structural and Technological Changes

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# OUTLINE

- Introduction
- Mikhalevich models
- Optimisation tools
- Software realization
- Perspectives

# Introduction

Transition from the planned socialist economics to the market one

- less regulated
- more flexible
- more effective

# Introduction

Transition from the planned socialist economics to the market one

- less regulated
- more flexible
- more effective  $\implies$  In what sense?

# Mikhalevich models



Corresponding Member of the NAS of Ukraine  
M.V. Mikhalevich (11.08.1956 – 14.12.2009)

# Mikhalevich models

How to determine the structural and technological changes that would reduce the costs of production and thus would increase the incomes of ultimate customers and make the economy more dynamic.

How to select or adjust technical coefficients to improve properties of the economic process.

These models of M. V. Mikhalevich can be called inverse models of the Leontief type.

# Mikhalevich models

SERGIENKO I.V., MIKHALEVICH M.V.,  
STETSYUK P.I., KOSHLAI L.B.

Intersectoral model for planning structural and technological reorganization

Cybernetics and System Analysis. – 1998. – № 3. – P. 3–17.  
(Engl. transl.: 1998. – 34(3). – P. 319–330)

SERGIENKO I.V., MIKHALEVICH M.V.,  
STETSYUK P.I., KOSHLAI L.B.

Models and informational technologies for decision support under structural and technological reorganization

Cybernetics and System Analysis. – 2009. – № 2. – P. 26–49.  
(Engl. transl.: 2009. – 45(2). – P. 187–203)

# Mikhalevich models

SERGIENKO, I.V., MIKHALEVICH, M.V., KOSHLAI, L.B.  
Optimization Models in a Transitional Economy. –  
Springer Optimization and its Application, Vol. 101, 2014, VIII,  
334 p.



# Mikhalevich models

Problem A: to maximize the total income of ultimate customers

Problem B: to maximize the multiplier  
„increase of incomes – increase of production“

# Notations

Let an economy contain  $N$  pure industries manufacturing only one type of products;  $i, j = \overline{1, N}$  be the numbers of these branches.

Denote by  $a_{ij}$  the value of direct production costs of the branch  $i$  for manufacturing a unit of production of the branch  $j$ .

This quantity can be expressed in both natural and cost measures depending on the information available. The matrix  $A = \{a_{ij}\}_{i,j=\overline{1,N}}$  is a matrix of the coefficients of direct costs.

Final product  $y_i$  and gross output  $x_i$  in every branch are related:

$$x_i = \sum_{j=1}^N a_{ij}x_j + y_i, \quad i = \overline{1, N}$$

i.e.

$$y = (E - A)x \quad \text{or} \quad x = (E - A)^{-1}y.$$

$E$  is  $(N \times N)$ -identity matrix.

# Assumptions

Let  $q_i$  be a share of ultimate incomes  
(payment for labor, social transfer, and profit)  
in the price of the production of the branch  $i$ ,  $q = \{q_i\}_{i=\overline{1,N}}$ ,  
 $x_i$  be the gross output of this branch,  $x = \{x_i\}_{i=\overline{1,N}}$ .

It is supposed that final incomes of consumers are proportional to gross output

$$D = \sum_{i=1}^N q_i x_i = \langle q, x \rangle.$$

# Assumptions

It is supposed that final output  $y_i$  in each branch consists of two parts:

the first being proportional to ultimate incomes  $D$ , and  
the second being independent of  $D$ :

$$y_i = \alpha_i D + h_i, \quad i = \overline{1, N},$$

coefficients  $\alpha_i$  characterise the structure of individual consumption and internal investments, and

coefficients  $h_i$  are determined from the export/import balance of branches and the structure of public consumption,

$$\alpha = \{\alpha_i\}_{i=\overline{1, N}}, \quad h = \{h_i\}_{i=\overline{1, N}}.$$

# Assumptions

One more assumption of the model is a linear relationship between the share of the added cost  $\tilde{q}_i$  in the price the product of the branch  $i$  and the share of ultimate incomes in the price of this product

$$\tilde{q}_i = l_i q_i + d_i, i = \overline{1, N},$$

where  $l_i$  is a fiscal multiplier of ultimate incomes, and  $d_i$  is the share of other components of the added cost in the price of the product of the  $i$ th branch.

If we substitute

$$x = (E - A)^{-1}y$$

and

$$y_i = \alpha_i D + h_i, \quad i = \overline{1, N}$$

into

$$D = \sum_{i=1}^N q_i x_i,$$

we get

$$D = \sum_{i=1}^N q_i x_i = \langle q, x \rangle = \langle q, (E - A)^{-1}y \rangle,$$

$$D = \frac{q^T (E - A)^{-1} h}{1 - q^T (E - A)^{-1} \alpha}.$$

# Notations

$z^T = q^T (E - A)^{-1}$  is a vector of additional variables characterizing the structure of final incomes from different economic activities

$A = \{a_{ij}\}_{i,j=\overline{1,N}}$  – Leontief matrix of direct costs for  $N$  branches  
 $\Delta A = \{\Delta a_{ij}\}_{i,j=\overline{1,N}}$  defines changes of entries in it

$q = \{q_i\}_{i=\overline{1,N}}$  is a part of final incomes in every branch  
 $\Delta q = \{\Delta q_i\}_{i=\overline{1,N}}$  defines changes of entries in this vector



# Problem A

maximize the total income of ultimate consumers

$$F_1(z) = \frac{z^T h}{1 - z^T \alpha} \rightarrow \max \quad (1A)$$

# Problem B

maximize the multiplier

„increase of incomes – increase of production“

$$F_2(z) = z^T \alpha \rightarrow \max \quad (1B)$$

# Constraints

$$z_j - \sum_{i=1}^N (a_{ji} + \Delta a_{ji}) z_i = q_j + \Delta q_j, \quad j = \overline{1, N} \quad (2)$$

(it defines additional variables)

$$\begin{aligned} & \beta(a_{jj} + \Delta a_{jj}) + \beta(l_j(q_j + \Delta q_j) + d_j) + \\ & + \sum_{i=1, i \neq j}^N (a_{ji} + \Delta a_{ij}) \leq \beta, \quad j = \overline{1, N} \end{aligned} \quad (3)$$

(the constraints that exclude the intensification of the inflation of costs,  $0 < \beta < 1$  is a given confidence parameter)

# Constraints

$$a_{jj} + \Delta a_{jj} + l_j(q_j + \Delta q_j) + d_j \leq 1 \quad j = \overline{1, N} \quad (4)$$

(balance of expences and value added)

$$\underline{\Delta q_i} \leq \Delta q_i \leq \overline{\Delta q_i}, \quad \underline{\Delta a_{ij}} \leq \Delta a_{ij} \leq \overline{\Delta a_{ij}}, \quad j = \overline{1, N} \quad (5)$$

(bounds for coefficients)

# Constraints

$$\sum_{j=1}^N \sum_{i=1}^N b_{kij} \max\{0, -\Delta a_{ij}\} \leq B_k \quad k = \overline{1, K} \quad (6)$$

(resource constraints:  $K$  is the number of resources,  $B_k$  is the volume of the  $k$ th resource intended to carry out structural and technological changes,  $b_{kij}$  is the expenditure of this resource in taking the measures that provide a unitary decrease in the expenses of the production of the branch  $i$  to produce a unit of production of the branch  $j$ ).

## Constraints, once more

$$z_j - \sum_{i=1}^N (a_{ji} + \Delta a_{ji}) z_i = q_j + \Delta q_j, \quad j = \overline{1, N} \quad (2)$$

$$\begin{aligned} & \beta(a_{jj} + \Delta a_{jj}) + \beta(l_j(q_j + \Delta q_j) + d_j) + \\ & + \sum_{i=1, i \neq j}^N (a_{ji} + \Delta a_{ji}) \leq \beta, \quad j = \overline{1, N} \end{aligned} \quad (3)$$

$$a_{jj} + \Delta a_{jj} + l_j(q_j + \Delta q_j) + d_j \leq 1 \quad j = \overline{1, N} \quad (4)$$

$$\underline{\Delta q_i} \leq \Delta q_i \leq \overline{\Delta q_i}, \quad \underline{\Delta a_{ij}} \leq \Delta a_{ij} \leq \overline{\Delta a_{ij}}, \quad i, j = \overline{1, N} \quad (5)$$

$$\sum_{j=1}^N \sum_{i=1}^N b_{kij} \max\{0, -\Delta a_{ij}\} \leq B_k \quad k = \overline{1, K} \quad (6)$$

# Optimisation

- Large scale
- Non-convex
- Non-smooth
- Complex feasible set
- Local extrema

# Shor's algorithms



Academician N.Z. Shor (01.01.1937 - 25.02.2006)



# Shor's algorithms

First the method of generalised gradient descent was proposed in 1962.

In 1969–1971 new methods with space transformations were proposed.

This permits to solve problems with non-smooth ravine-like functions.

# Shor's algorithms

The most known particular cases of subgradient methods with space transformation:

- Ellipsoid Method. Some polynomial algorithms were developed with its use for some convex and discrete programming problems.
- $r$ -algorithms. One of the most efficient tools to solve non-smooth optimisation problems. In smooth case they are comparable with the most efficient realisations of Conjugate Directions Methods and Quasi-Newton Methods.

# Shor's algorithms

SHOR, N. Z.,  
Minimization Methods for Non-Differentiable Functions,  
Springer-Verlag, Berlin, 199 p. (1985)

# Software realizations: IOMSTC (MiSTC)

IOMSTC – Intersectoral Optimisation Models of Structural and Technological Changes

MiSTC – Mikhalevich Structural and Technological Changes

# Software realizations: IOMSTC (MiSTC)

IOMSTC consists of the user interface, base of models, and a set of program modules implementing optimisation algorithms:

- maximization of ultimate incomes of consumers;
- maximization of the multiplier “increase of incomes–increase of production”;
- checking the compatibility of the system of constraints.

# Software realization: IOMSTC (MiSTC)

- Numerical procedure:  $r$ -algorithms and the use of non-smooth penalty functions.
- Multistart to overcome existence of multiple local extrema.
- Convenient interface.
- Data preparation tools.
- Preparation of reports in html-format.

# Software realization: IOMSTC (MiSTC)

The screenshot displays the MiSTC software interface. The window title is "MiSTC - [C:/SHARE/Conferences/Кишинев2014/MiSTC/model3.xml]". The menu bar includes "Файл", "Язык", "Проект", "Окно", and "Справка". The main workspace is divided into several tabs: "Основные параметры", "Матрица прямых затрат", "Нижняя граница", "Верхняя граница", "Вектора переменных", "Ресурсы", "Векторы", "Редактировать", and "Результаты".

The "Основные параметры" tab is active, showing the following fields:

- Название проекта: Project 1
- Количество отраслей: 7
- Инфляционный параметр: 0.9500
- Количество стартовых точек: 2

Below these fields, the "Целевая функция" (Objective Function) section contains three radio buttons:

- Доход потребителей
- Мультипликатор "затраты-выпуск"
- Проверка совместности систем ограничений

On the right side, the "Ресурсы" (Resources) tab is active, displaying a table with the following data:

Название ресурса	
1	Топливо-энергетический комп...
2	Тяжелая индустрия (производств...
3	Легкая промышленность (произ...
4	Агропромышленный комплекс
5	Строительство
6	Транспорт и связь
7	Другие отрасли сферы услуг

# Software realization: IOMSTC (MiSTC)

MiSTC - [C:/SHARE/Conferences/Кишинев2014/MiSTC/model3.xml]

Файл Язык Проект Окно Справка

Основные параметры Матрица прямых затрат Нижняя граница Верхняя граница Вектора переменных Ресурсы Векторы Редактировать Результаты

	1	2	3	4
<b>Топливо-энергетический комплекс</b>	0.1685	0.0230002	0.0815	0.0
Тяжелая индустрия (производство средств производства)	0.0695	0.1255	0.088	0.0
Легкая промышленность (производство непродовольственных товаров)	0.117316	0.2685	0.191	0.0
Агропромышленный комплекс	0.127	0.1335	0.0969999	0.0
Строительство	0.146	0.0284997	0.103	0.0
Транспорт и связь	0.112	0.196499	0.0519339	0.0

1		1	
1 Рекордная целевая функция	1.9120	1 Время выполнения программы (сек)	2.6581
2 Максимальная невязка в рекордной точке	8.3892	2 Останов	по ар
3 Целевая функция в конечной точке	1.9120	3 Количество итераций	4544
4 Максимальная невязка в конечной точке	2.4104	4 Количество вычислений функционала	7153

Удалить одинаковые

Критерий  
0.0

Удалить

Кoeffициенты  
 Все  
 Изменения

Восстановить

Выделение



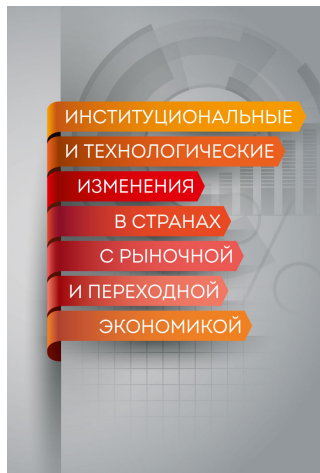
# Project

"Analysis of Institutional and Technological Changes  
in Market and Transition Economies  
on the Background of the Present Financial Crisis"

Project funded by Swiss National Scientific Foundation  
Number of the JRP: SNSF IZ73ZO 127962  
Duration: 01.01.2010-31.12.2012

Co-ordinator: Dr. Jean-Francois Emmenegger  
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Ukrainian team leader: Dr. Petro Stetsyuk  
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<http://www.icyb.kiev.ua/file/120/books/Stetsyuk-Editor-2015.pdf>

# Perspectives

- Data – calculations
- New formulations of optimisation models
- Other forms of collaboration...

Vielen Dank  
für Ihre Aufmerksamkeit!

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