Optimisation Problems for Planning Structural and Technological Changes

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Introduction

Transition from the planned socialist economics to the market one

- less regulated
- more flexible
- more effective

Introduction

Transition from the planned socialist economics to the market one

- less regulated
- more flexible
- more effective $\Longrightarrow$ In what sense?

Mikhalevich models

Corresponding Member of the NAS of Ukraine
M.V. Mikhalevich (11.08.1956 – 14.12.2009)

Mikhalevich models

How to determine the structural and technological changes that would reduce the costs of production and thus would increase the incomes of ultimate customers and make the economy more dynamic.

How to select or adjust technical coefficients to improve properties of the economic process.

These models of M. V. Mikhalevich can be called inverse models of the Leontief type.

Mikhalevich models

Sergienko I.V., Mikhalevich M.V.,
Stetsyuk P.I., Koshlai L.B.
Intersectoral model for planning structural and technological reorganization
Cybernetics and System Analysis. – 1998. – № 3. – P. 3–17.
(Engl. transl.: 1998. – 34(3). – P. 319–330)

Sergienko I.V., Mikhalevich M.V.,
Stetsyuk P.I., Koshlai L.B.
Models and informational technologies for decision support under structural and technological reorganization
Cybernetics and System Analysis. – 2009. – № 2. – P. 26–49.
(Engl. transl.: 2009. – 45(2). – P. 187–203)

Mikhalevich models

Sergienko, I.V., Mikhalevich, M.V., Koshlai, L.B.
Optimization Models in a Transitional Economy. –
Springer Optimization and its Application, Vol. 101, 2014, VIII,
334 p.

Mikhalevich models

Problem A: to maximize the total income of ultimate customers

Problem B: to maximize the multiplicator
„increase of incomes – increase of production“

Notations

Let an economy contain N pure industries manufacturing only one type of products; $i, j = \overline{1, N}$ be the numbers of these branches.

Denote by a_{ij} the value of direct production costs of the branch i for manufacturing a unit of production of the branch j.

This quantity can be expressed in both natural and cost measures depending on the information avaliable. The matrix $A = \{a_{ij}\}_{i,j=\overline{1,N}}$ is a matrix of the coefficients of direct costs.

Final product y_i and gross output x_i in every branch are related:

$$x_i = \sum_{j=1}^{N} a_{ij} x_j + y_i, \quad i = \overline{1, N}$$

i.e.

$$y = (E - A)x \quad \text{or} \quad x = (E - A)^{-1} y.$$

E is $(N \times N)$-identity matrix.

Assumptions

Let q_i be a share of ultimate incomes
(payment for labor, social transfer, and profit)
in the price of the production of the branch i, $q = \{q_i\}_{i=\overline{1,N}}$,
x_i be the gross output of this branch, $x = \{x_i\}_{i=\overline{1,N}}$.

It is supposed that final incomes of consumers are proportional to gross output

$$D = \sum_{i=1}^{N} q_i x_i = \langle q, x \rangle .$$

Assumptions

It is supposed that final output y_i in each branch consists of two parts:
the first being proportional to ultimate incomes D, and
the second being independent of D:

$$y_i = \alpha_i D + h_i, \quad i = \overline{1, N},$$

coefficients α_i characterise the structure of individual consumption and internal investments, and
coefficients h_i are determined from the export/import balance of branches and the structure of public consumption,
$\alpha = \{\alpha_i\}_{i=\overline{1,N}}$, $h = \{h_i\}_{i=\overline{1,N}}$.

Assumptions

One more assumption of the model is a linear relationship between the share of the added cost $\tilde{q}_i$ in the price the product of the branch i and the share of ultimate incomes in the price of this product

$$\tilde{q}_i = l_i q_i + d_i, i = \overline{1, N},$$

where l_i is a fiscal multiplier of ultimate incomes, and d_i is the share of other components of the added cost in the price of the product of the ith branch.

If we substitute

$$x = (E - A)^{-1} y$$

and

$$y_i = \alpha_i D + h_i, \quad i = \overline{1, N}$$

into

$$D = \sum_{i=1}^{N} q_i x_i,$$

we get

$$D = \sum_{i=1}^{N} q_i x_i = \langle q, x \rangle = \left\langle q, (E - A)^{-1} y \right\rangle,$$

$$D = \frac{q^T (E - A)^{-1} h}{1 - q^T (E - A)^{-1} \alpha}.$$

Notations

$z^T = q^T(E - A)^{-1}$ is a vector of additional variables characterizing the structure of final incomes from different economic activities

$A = \{a_{ij}\}_{i,j=\overline{1,N}}$ – Leontief matrix of direct costs for N branches
$\Delta A = \{\Delta a_{ij}\}_{i,j=\overline{1,N}}$ defines changes of entries in it

$q = \{q_i\}_{i=\overline{1,N}}$ is a part of final incomes in every branch
$\Delta q = \{\Delta q_i\}_{i=\overline{1,N}}$ defines changes of entries in this vector

Problem A

maximize the total income of ultimate consumers

$$F_1(z) = \frac{z^T h}{1 - z^T \alpha} \to \max \qquad (1A)$$

Problem B

maximize the multiplicator
„increase of incomes – increase of production“

$$F_2(z) = z^T \alpha \rightarrow \max \qquad (1B)$$

Constraints

$$z_j - \sum_{i=1}^{N}(a_{ji} + \Delta a_{ji})z_i = q_j + \Delta q_j, \quad j = \overline{1, N} \tag{2}$$

(it defines additional variables)

$$\beta(a_{jj} + \Delta a_{jj}) + \beta(l_j(q_j + \Delta q_j) + d_j) +$$

$$+ \sum_{i=1, i\neq j}^{N} (a_{ji} + \Delta a_{ij}) \leq \beta, \quad j = \overline{1, N} \tag{3}$$

(the constraints that exclude the intensification of the inflation of costs, $0 < \beta < 1$ is a given confidence parameter)

Constraints

$$a_{jj} + \Delta a_{jj} + l_j(q_j + \Delta q_j) + d_j \leq 1 \quad j = \overline{1, N} \tag{4}$$

(balance of expences and value added)

$$\underline{\Delta q_i} \leq \Delta q_i \leq \overline{\Delta q_i}, \quad \underline{\Delta a_{ij}} \leq \Delta a_{ij} \leq \overline{\Delta a_{ij}}, \quad j = \overline{1, N} \tag{5}$$

(bounds for coefficients)

Constraints

$$\sum_{j=1}^{N}\sum_{i=1}^{N} b_{kij}\max\{0,-\Delta a_{ij}\} \le B_k \quad k=\overline{1,K} \tag{6}$$

(resource constraints: K is the number of resources, B_k is the volume of the kth resource intended to carry out structural and technological changes, b_{kij} is the expenditure of this resource in taking the measures that provide a unitary decrease in the expenses of the production of the branch i to produce a unit of production of the branch j).

Constraints, once more

$$z_j - \sum_{i=1}^{N}(a_{ji} + \Delta a_{ji})z_i = q_j + \Delta q_j, \quad j = \overline{1, N} \tag{2}$$

$$\beta(a_{jj} + \Delta a_{jj}) + \beta(l_j(q_j + \Delta q_j) + d_j) + \\ + \sum_{i=1, i\neq j}^{N}(a_{ji} + \Delta a_{ij}) \leq \beta, \quad j = \overline{1, N} \tag{3}$$

$$a_{jj} + \Delta a_{jj} + l_j(q_j + \Delta q_j) + d_j \leq 1 \quad j = \overline{1, N} \tag{4}$$

$$\underline{\Delta q_i} \leq \Delta q_i \leq \overline{\Delta q_i}, \quad \underline{\Delta a_{ij}} \leq \Delta a_{ij} \leq \overline{\Delta a_{ij}}, \quad i, j = \overline{1, N} \tag{5}$$

$$\sum_{j=1}^{N}\sum_{i=1}^{N} b_{kij} \max\{0, -\Delta a_{ij}\} \leq B_k \quad k = \overline{1, K} \tag{6}$$

Optimisation

- Large scale
- Non-convex
- Non-smooth
- Complex feasible set
- Local extrema

Shor's algorithms

Academician N.Z. Shor (01.01.1937 - 25.02.2006)

Shor's algorithms

First the method of generalised gradient descent was proposed in 1962.

In 1969–1971 new methods with space transformations were proposed.

This permits to solve problems with non-smooth ravine-like functions.

Shor's algorithms

The most known particular cases of subgradient methods with space transformation:

- Ellipsoid Method. Some polynomial algorithms were developed with its use for some convex and discrete programming problems.
- r–algorithms. One of the most efficient tools to solve non-smooth optimisation problems. In smooth case they are comparable with the most efficient realisations of Conjugate Directions Methods and Quasi-Newton Methods.

Shor's algorithms

Shor, N. Z.,
Minimization Methods for Non-Differentiable Functions,
Springer-Verlag, Berlin, 199 p. (1985)

Software realizations: IOMSTC (MiSTC)

IOMSTC – Intersectoral Optimisation Models of Structural and Technological Changes

MiSTC – Mikhalevich Structural and Technological Changes

Software realizations: IOMSTC (MiSTC)

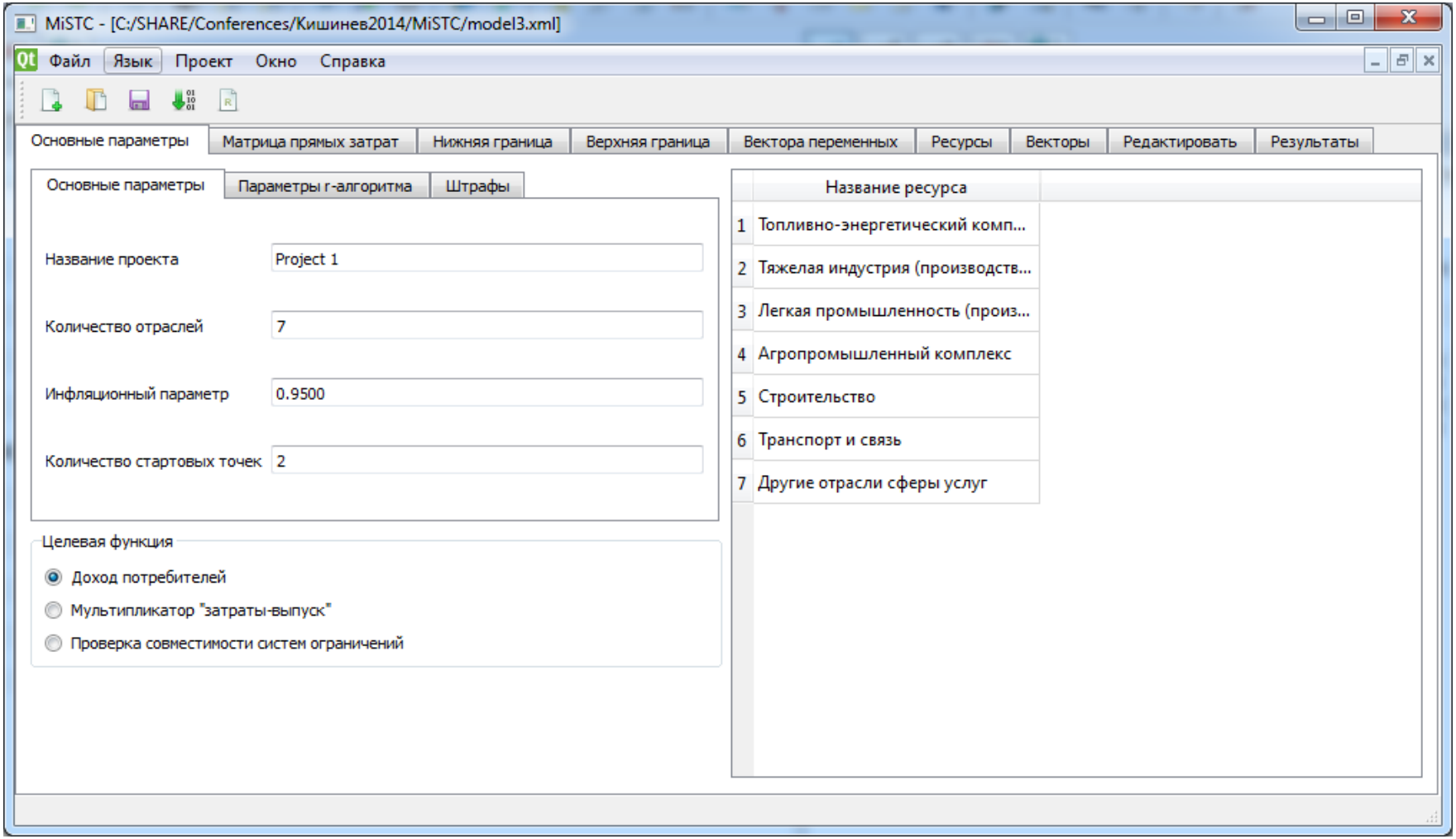
IOMSTC consists of the user interface, base of models, and a set of program modules implementing optimisation algorithms:

- maximization of ultimate incomes of consumers;
- maximization of the multiplier “increase of incomes–increase of production”;
- checking the compatibility of the system of constraints.

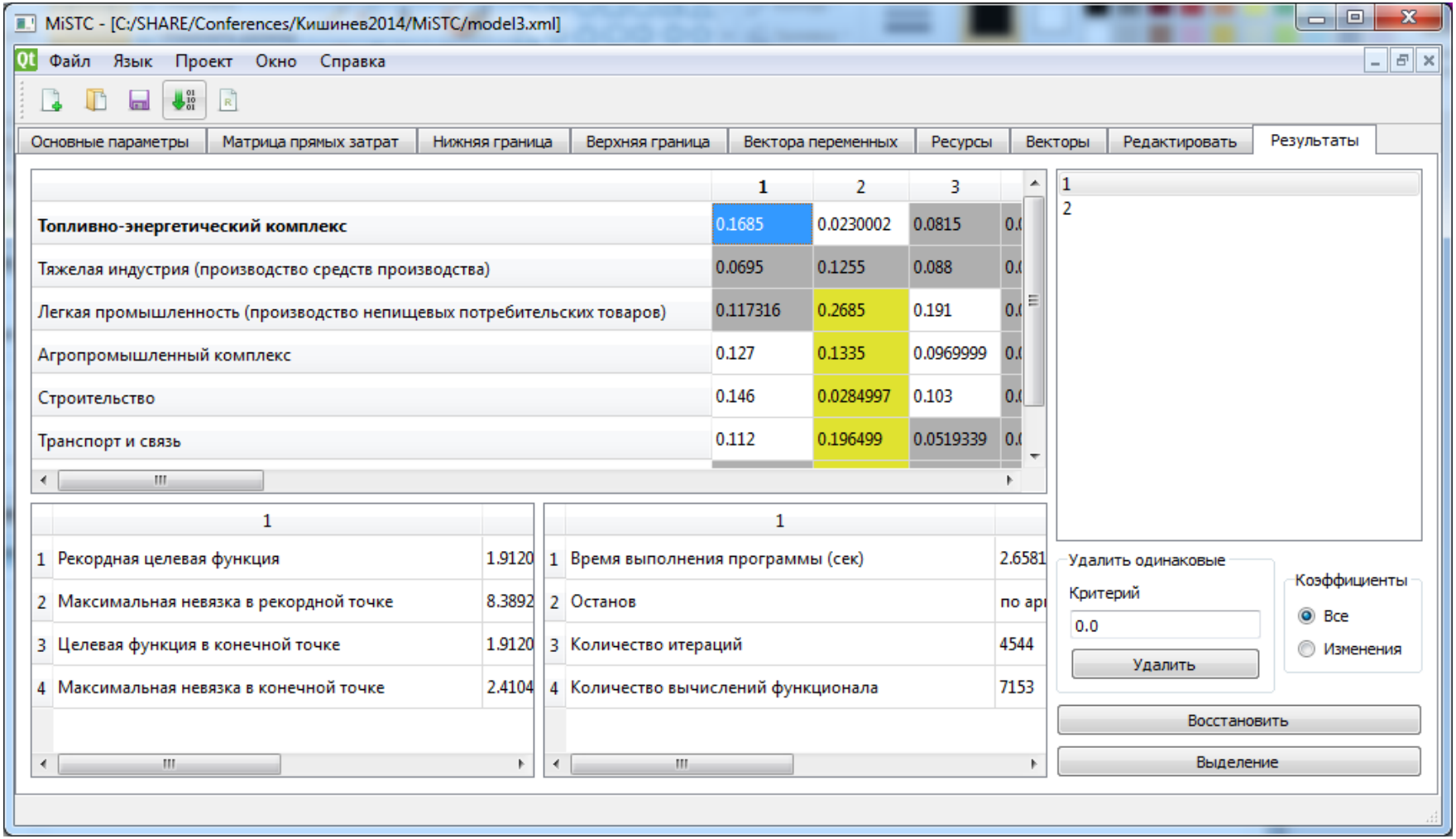
Software realization: IOMSTC (MiSTC)

- Numerical procedure: r–algorithms and the use of non-smooth penalty functions.
- Multistart to overcome existence of multiple local extrema.
- Convenient interface.
- Data preparation tools.
- Preparation of reports in html-format.

Software realization: IOMSTC (MiSTC)



Software realization: IOMSTC (MiSTC)



Project

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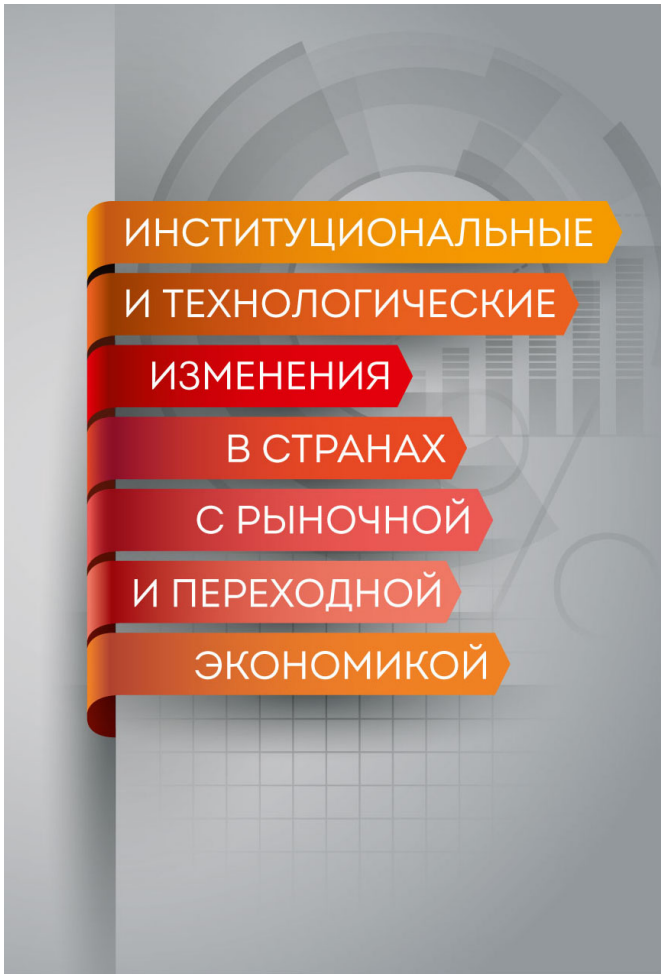
Book


ИНСТИТУЦИОНАЛЬНЫЕ
И ТЕХНОЛОГИЧЕСКИЕ
ИЗМЕНЕНИЯ
В СТРАНАХ
С РЫНОЧНОЙ
И ПЕРЕХОДНОЙ
ЭКОНОМИКОЙ

Book

Stetsyuk, P. (Ed.), Bortis, H., Emmenegger, J.-F., Koshlai, L., Beresovskyi, O., Bardadym, T., Pervukhina, E., Golikova, V., Osipov, K., Karpets E., Pylypovskyi, O.
Institutional and Technological Changes in Market and Transitional Economies. –
Publishing House „Kyiv Mohyla Academy“, Kyiv, 336 p. (2015) (In Russian),

http://www.icyb.kiev.ua/file/120/books/Stetsyuk-Editor-2015.pdf

Perpectives

- Data – calculations
- New formulations of optimisation models
- Other forms of collaboration...

Vielen Dank für Ihre Aufmerksamkeit!

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