# *Using the FLQ formula in estimating interregional output multipliers*

9. Input-Output Workshop, Bremen
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# Content

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# Data and literature

Data: Survey-based interregional IO table for South Korea (28 sectors, 16 regions) from 2005 (collected by the Bank of Korea):

| | | region 1 | | | region 2 | | | ... |
|---|---|---|---|---|---|---|---|---|
| | | sector 1 | sector 2 | sector 3 | sector 1 | sector 2 | sector 3 | |
| region 1 | sector 1 | | | | | | | |
| | sector 2 | | | | | | | |
| | sector 3 | | | | | | | |
| region 2 | sector 1 | | | | | | | |
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Relevant recent literature:

- Flegg, Tohmo (2018): The regionalization of national input-output tables: a study of South Korean regions, to appear in *Papers in Regional Science*
- Jahn (2016): Extending the FLQ formula: a location quotient-based interregional input–output framework, *Regional Studies* 51(10)

# Intraregional input coefficients and multipliers

Reminder: Intraregional input-coefficient matrices $A^{rr}$ are defined as:

$$A^{rr} = (a_{ij}^{rr}) = (z_{ij}^{rr}/x_j^r) \text{ for } r = 1, \ldots, R \tag{1}$$

Notation:

| | |
|---|---|
| $z_{ij}^{sr}$ | IO transactions from sector $i$ in region $s$ to sector $j$ in region $r$ |
| $x_j^r$ | output of sector $j$ in region $r$ |

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FLQ estimate for the **intra**regional input coefficients:

$$\hat{a}_{ij}^{rr} = \min(a_{ij}, a_{ij} \cdot FLQ_{ij}^r) = \min(a_{ij}, a_{ij} \cdot CILQ_{ij}^r \cdot \lambda^r) \qquad (2)$$

Notation:

| | |
|---|---|
| $a_{ij}$ | national input coefficient (without imports, $a_{ij} = {}^{z_{ij}}/{}_{x_j}$) |
| $CILQ_{ij}^r$ | cross-industry location quotient $\left( CILQ_{ij}^r = \frac{\epsilon_i^r/\epsilon_j^r}{\epsilon_i/\epsilon_j} \text{ for } i \neq j \right)$ |
| $\epsilon_j^r$ | size of sector $j$ in region $r$ (often employment or output) |
| $\lambda^r$ | adjustment for size of region $r$ $\left( \lambda^r = [\log_2(1 + {}^{\epsilon^r}/{}_{\epsilon})]^{\delta} \right)$ |

# Intraregional multipliers

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Intraregional Leontief inverse:

$$L^r = (I - A^{rr})^{-1} \tag{3}$$

Intraregional output multiplier:

$$N^r_j = \sum_i L^r_{(ij)} \tag{4}$$

Mathematically, the multiplier is the sum of the column of the Leontief inverse which refers to the respective industry.

# South Korean IRIO table: Previous results

Performance of models for the intraregional output multipliers with different specifications of $\delta$ (region- and/or sector-specific) in the FLQ formula:

| method | MAPE | error var. | BIC | AIC | k | N |
|---|---|---|---|---|---|---|
| SLQ | 27.287 | 0.190 | -743.4 | -743.4 | 0 | 448 |
| CILQ | 23.045 | 0.117 | -960.0 | -960.0 | 0 | 448 |
| FLQ with $\delta$ | 8.028 | 0.017 | -1807.4 | -1811.5 | 1 | 448 |
| FLQ with $\delta_r$ | 7.247 | 0.015 | -1789.4 | -1855.1 | 16 | 448 |
| FLQ with $\delta_j$ | 6.401 | 0.011 | **-1857.1** | **-1972.0** | 28 | 448 |
| FLQ with $\delta_{jr}$ | **1.805** | **0.003** | 154.3 | -1684.6 | 448 | 448 |

Information criteria *BIC* and *AIC* used to compare error variances for models with different numbers of parameters $k$

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Starting with the interregional coefficient matrices

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one way to define the interregional Leontief inverse is:

$$L = (I - A)^{-1} \text{ with} \tag{6}$$

$$A = \begin{pmatrix} A^{11} & \ldots & A^{1R} \\ \vdots & \ddots & \vdots \\ A^{R1} & \ldots & A^{RR} \end{pmatrix} \tag{7}$$

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Denoting the blocks of $L$ with $L^{sr}$ (similarly to $A$), the interregional multipliers are defined as $M_j^r = \sum_{i,s} L_{(ij)}^{sr}$.

# Empirical importance of interregional IO transactions

Intra- and interregional multipliers for 4 of 16 South Korean regions (2005) for 28 sectors:

| | inter | | intra | | ratio (inter/intra) | |
|---|---|---|---|---|---|---|
| region | mean | std. dev. | mean | std. dev. | mean | std. dev. |
| Seoul | 1.998 | 0.341 | 1.274 | 0.123 | 1.582 | 0.313 |
| Busan | 1.982 | 0.332 | 1.277 | 0.105 | 1.551 | 0.219 |
| Gangwon-do | 1.961 | 0.318 | 1.266 | 0.134 | 1.555 | 0.239 |
| Jeju-do | 1.925 | 0.360 | 1.195 | 0.104 | 1.614 | 0.285 |
| total | 1.961 | | 1.249 | | 1.572 | |

Seoul: north-west coast, Busan: south-east coast, Gangwon-do: mountainous border region in the north, Jeju-do: remote island

# Estimation: Gravity model

In order to estimate interregional IO transactions, a gravity model can be used (cf. Jahn, 2016):

$$\ln z_{ij}^{sr} = \beta_0 + \beta_1 \ln x_i^s + \beta_2 \ln x_j^r + \beta_3 \ln d_{sr} + \beta_4 border_{sr} + \eta_{ij}^{sr} \quad \text{for } s \neq r$$

Notation:

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|---|---|
| $z_{ij}^{sr}$ | IO transactions from sector $i$ in region $s$ to sector $j$ in region $r$ |
| $x_i^s$ | output of sending sector |
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Simple alternative ignoring spatial information (cf. Batten, 1982):
$\ln z_{ij}^{sr} = \beta_0 + \beta_1 \ln x_i^s + \beta_2 \ln x_j^r + \eta_{ij}^{sr}$

# South Korea: Gravity model results

| variable | coeff. | robust se | t value | [95% | conf.int.] |
|---|---|---|---|---|---|
| output: sending sec. | 0.836 | 0.003 | 261.42 | 0.829 | 0.842 |
| output: receiving sec. | 0.899 | 0.003 | 259.94 | 0.892 | 0.906 |
| distance | -0.626 | 0.015 | -42.33 | -0.655 | -0.597 |
| adjacency | 0.075 | 0.021 | 3.61 | 0.034 | 0.116 |
| constant | -17.552 | 0.110 | -159.02 | -17.769 | -17.336 |
| $R^2$ | 0.469 | | | | |
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- All coefficients have the expected sign and magnitude
- Alternative definitions of "distance" don't affect the results
- The simple model has only slightly less explanatory power ($R^2 = 0.457$)

# The FLQ formula in the interregional setting

Reminder: Estimates of intraregional input coefficients according to the FLQ formula:

$$\hat{a}_{ij}^{rr} = \min(a_{ij}, a_{ij} \cdot FLQ_{ij}^{r}) \tag{8}$$

This yields in absolute terms: $\hat{z}_{ij}^{rr} = \hat{a}_{ij}^{rr} \cdot x_j^r$, where $x_j^r$ is either known or estimated (usually via $\epsilon_j^r$).

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Since national transactions $z_{ij}$ are also known, the amount of interregional transactions consistent with the FLQ formula is given by:

$$\varepsilon_{ij}^{FLQ} = z_{ij} - \sum_r \hat{z}_{ij}^{rr} \geq 0 \quad (9)$$

The non-negativity of this FLQ-residual follows from the fact that $\hat{a}_{ij}^{rr} \leq a_{ij}$ (proof: Jahn, 2016).

# Consistent estimation of IRIO transactions

Combining intraregional (FLQ) and interregional (gravity) estimates, we get as initial estimates:

$$\hat{z}_{ij}^{sr} = \begin{cases} \min(a_{ij}, a_{ij} \cdot FLQ_{ij}^{r}) \cdot x_j^r \quad \text{for} \quad s = r \\ c_{ij} \cdot \left( (x_i^s)^{\hat{\beta}_1} \cdot (x_j^r)^{\hat{\beta}_2} \cdot (d_{sr})^{\hat{\beta}_3} \cdot \exp(\hat{\beta}_4 border_{sr}) \right) \quad \text{for} \quad s \neq r. \end{cases}$$

The constant $c_{ij}$ is chosen such that $\sum_{s,r(s \neq r)} \hat{z}_{ij}^{sr} = \varepsilon_{ij}^{FLQ}$.

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The constant $c_{ij}$ is chosen such that $\sum_{s,r(s \neq r)} \hat{z}_{ij}^{sr} = \varepsilon_{ij}^{FLQ}$.

In order to obtain the final estimates, the following optimization problem is solved:

$$\min_{\bar{z}_{ij}^{sr}} S = \sum_{i,j,s,r} \frac{(\bar{z}_{ij}^{sr} - \hat{z}_{ij}^{sr})^2}{\bar{z}_{ij}^{sr}} \qquad (10)$$

subject to $\sum_{i,s} \bar{z}_{ij}^{sr} \leq x_j^r$ and $\sum_{s,r} \bar{z}_{ij}^{sr} = z_{ij}$. The FLQ-based intraregional estimates are kept fix $\bar{z}_{ij}^{rr} = \hat{z}_{ij}^{rr}$ in this optimization.

# South Korea: New results

Performance of models for the **inter**regional output multipliers with different specifications of $\delta$ in the FLQ formula:

| method | MAPE | error var. | BIC | AIC | k | N |
|---|---|---|---|---|---|---|
| FLQ $\delta$ + grav. | 9.168 | 0.066 | -1211.2 | -1215.3 | 1 | 448 |
| FLQ $\delta_j$ + grav. | 8.265 | 0.057 | -1114.7 | -1229.6 | 28 | 448 |
| FLQ $\delta_r$+ grav. | 8.495 | 0.060 | -1160.5 | -1226.2 | 16 | 448 |
| FLQ $\delta_{jr}$ + grav. | 5.630 | 0.041 | 1308.8 | -530.2 | 448 | 448 |
| FLQ $\delta$ + simple | 8.139 | 0.063 | **-1234.6** | -1238.7 | 1 | 448 |
| FLQ $\delta_j$ + simple | 7.385 | 0.053 | -1142.2 | -1257.2 | 28 | 448 |
| FLQ $\delta_r$ + simple | 7.670 | 0.054 | -1207.0 | **-1272.7** | 16 | 448 |
| FLQ $\delta_{jr}$ + simple | **5.043** | **0.039** | 1281.9 | -557.0 | 448 | 448 |
| CILQ + grav. | 9.310 | 0.068 | -1201.5 | -1201.5 | 0 | 448 |
| CILQ + simple | 9.280 | 0.069 | -1198.8 | -1198.8 | 0 | 448 |
| SLQ + grav. | 16.644 | 0.232 | -654.9 | -654.9 | 0 | 448 |
| SLQ + simple | 16.467 | 0.227 | -664.7 | -664.7 | 0 | 448 |

# Conclusions

The most important results:

- Ignoring interregional IO relations results in much too low multipliers

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- The penalization of the number of (free) parameters is important for model comparison

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- Ignoring interregional IO relations results in much too low multipliers
- Estimating interregional IO transactions requires a combination of intra- and interregional models, as well as a balancing algorithm
- The FLQ formula is also useful in estimating interregional multipliers
- The penalization of the number of (free) parameters is important for model comparison
- Ignoring spatial information does not lead to a worse approximation of true (interregional) multipliers

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